Disc 2/2/17

1.2.18 ✓

1.2.29

1.3.28 ✓

1.4.6 ✓

1.4.14

1.4.17 ✓

1.5.12 ✓

1.5.21

1.5.28

1.6.3 ✓

1.6.6

1.6.7

1.6.8

1.6.10

1.6.17

1.6.19

1.6.20

1.6.21

1.6.23

1.6.24

1.9.6 ✓

1.9.7

1.9.8

1.2.18 Let A be an $m \times n$ matrix and let c be a scalar. Show that if $cA = 0$, then $c = 0$ or $A = 0$.

Assume $cA = 0$. (To show if X, then Y, assume X, and then show Y must be true.)

Then for all i, j, $i \le m$, $j \le n$, $(cA)_{ij} = 0$.

But $(cA)_{ij} = c A_{ij}$ so $c A_{ij} = 0$.

If $c = 0$, then $c A_{ij} = 0$ for all $i \le m$, $j \le n$.

If $c \ne 0$, then $c A_{ij} = 0$ for all $i \le m$, $j \le n$

$\Rightarrow A_{ij} = 0$ for all $i \le m$, $j \le n$

$\Rightarrow A = 0$.

Thus either $c = 0$ or $A = 0$,

(To show X or Y, show $(\text{not } X) \Rightarrow Y$.)

1.3.28 Prove that if A is a regular 2×2 matrix, then its LU factorization is unique

(To show unique, show that $L_1 U_1 = A = L_2 U_2 \Rightarrow L_1 = L_2$ and $U_1 = U_2$)

Assume $L_1 U_1 = A = L_2 U_2$, L_1, L_2 are special lower (1's on diagonal) U_1, U_2 are upper triangles. (2×2)

$$\begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix} \begin{pmatrix} b & c \\ 0 & d \end{pmatrix} = \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix} \begin{pmatrix} j & k \\ 0 & l \end{pmatrix}$$

$$\begin{pmatrix} b & c \\ ab & ac+d \end{pmatrix} = \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} j & k \\ ij & ik+l \end{pmatrix}$$

WTS: $a = i$, $b = j$, $c = k$, $d = l$.

So, $b = j$, $c = k$, $ab = ij \Rightarrow a = i$ since $b = j$ and $ac + d = ik + l \Rightarrow d = l$ since $a = i$ and $c = k$.

Thus, $L_1 = L_2$, $U_1 = U_2$.

Hence the LU factorization of A is unique.

1.4.6. T or F: A singular matrix cannot be regular

T: A reg $\Rightarrow$ A nonsing

Thus A sing = A not nonsing $\Rightarrow$ A reg.

(Contrapositive.)

1.4.17. Justify the statement that there are $n!$ different permutation matrices.

In a perputation matrix there is a 1 in each row, and the 1 in each row is in a diff. column.

There are n possibilities for the 1 in the first row.

$n-1$ for the second (since 1 col. is taken up)

$n-2$ for the 3rd.

etc.

Thus there are $n(n-1)(n-2)\cdots = n!$ possibilities

1.5.12 Find all real 2×2 matrices that are their own inverses, i.e. $A = A^{-1}$.

$A = A^{-1}$ iff $AA = I \iff A^2 = I$.

Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix} = I$$

$$\iff \begin{matrix} a^2+bc = 1 & ab+bd = 0 \\ ac+cd = 0 & bc+d^2 = 1 \end{matrix}$$

$\iff ac = -cd, \quad ab = -bd$

$\iff a = -d, \quad a = -d,$

So $a^2 + bc = 1$

$bc + a^2 = 1,$

Thus $A = A^{-1}$ $\left(A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\right)$ iff

$a^2 + bc = 1$ and $a = -d$.

1.6.3 Show that $(AB)^T = A^T B^T$ iff A and B are square commuting matrices.

$\boxed{\Rightarrow}$ Assume $(AB)^T = A^T B^T$.

Then $B^T A^T = A^T B^T$

$\Rightarrow (B^T A^T)^T = (A^T B^T)^T$

$\Rightarrow AB = BA$. Thus A and B are square comm. matrices

$\boxed{\leftarrow}$ Assume A + B are square comm. matrices.

Then $(AB)^T = (BA)^T = A^T B^T$. ✓

1.9.6. For what values of a, b, c, is the matrix

$$A = \begin{pmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{pmatrix}$$ invertible?

$$\det A = -a\left(-a(0) - bc\right) + (-b)\left((-a)(-c) - b(0)\right)$$
$$= -a(-bc) - b(ac)$$
$$= abc - bac$$
$$= abc - abc, \quad a, b, c, \text{ are real \#s}$$
$$= 0.$$

Ans: A is never invertible.

- Hint for HW 1.6.28: Use Thm 1.29 on pg. 43

1.6.8 (a) Prove that $(AB)^{-T} = A^{-T} B^{-T}$.

Proof $(AB)^{-T} = ((AB)^{-1})^T$
$= (B^{-1}A^{-1})^T$
$= (A^{-1})^T (B^{-1})^T$
$= A^{-T} B^{-T}$

1.6.7
1.6.17